Mental Math with Tricks and Shortcuts

Addition

Technique: Add left to right

326 + 678 + 245 + 567 = 900, 1100, 1600, 1620, 1690, 1730, 1790, 1804, & **1816**

Note: Look for opportunities to combine numbers to reduce the number of steps to the solution. This was done with 6+8 = 14 and 5+7 = 12 above. Look for opportunities to form 10, 100, 1000, and etc. between numbers that are not necessarily next to each other. Practice!

Multiplication & Squaring

Some useful formulae

	Examples
$(a+b)^2 = a^2 + 2ab + b^2$	$49^2 = (40 + 9)^2 = 1600 + 720 + 81 =$ **2401**
$(a-b)^2 = a^2 - 2ab + b^2$	$56^2 = (60 - 4)^2 = 3600 - 480 + 16 =$ **3136**
$(a+b)(a-b) = a^2 - b^2$	$64 \times 56 = (60 - 4)(60 + 4) = 3600 - 16 =$ **3584**
$(a+b)(c+d) = (ac + ad) + (bc + bd)$	$23 \times 34 = (20 + 3)(30 + 4) = 600 + 80 + 90 + 12 =$ **782**
$(a+b)(c-d) = (ac - ad) + (bc - bd)$	$34 \times 78 = (30 + 4)(80 - 2) = 2400 - 60 + 320 - 8 =$ **2652**
$(a-b)(c-d) = (ac - ad) - (bc - bd)$	$67 \times 86 = (70 - 3)(90 - 4) = 6300 - 280 - 270 + 12 =$ **5762**
X = 1 to 9 & Y = Any Number	
$(X5)^2 = 100X(X+1) + 25$	$65^2 = 600(7) + 25 = 4200 + 25 =$ **4225**
$25 \times Y = (Y \times 100)/4$	$25 \times 76 = 7600/4 =$ **1900**
$50 \times Y = (Y \times 100)/2$	$50 \times 67 = 6700/2 =$ **3350**
$75 \times Y = 3(Y \times 100)/4$	$75 \times 58 = (5800 \times 3)/4 = 17400/4 =$ **4350**

Square any Two Digit Number (a = 10's digit & b = 1's digit)

$(ab)^2 = 100a^2 + 20(a \times b) + b^2$ $\qquad 67^2 = 100(36) + 20(42) + 49 =$ **4489**

Multiply any Two 2 Digit Numbers (a & c = 10's digit, b & d = 1'a digit)

$ab \times cd = 100(a \times c) + 10[(b \times c) + (a \times d)] + (b \times d)$ $\qquad 53 \times 68 = 3000 + 580 + 24 =$ **3604**

Tricks using $(X5)^2$

$(X5 - a)^2 = (X5)^2 - X5(2a) + a^2)$ $\qquad 63^2 = (65 - 2)^2 = 4225 - 260 + 4) =$ **3969**

$(X5 + a)^2 = (X5)^2 + X5(2a) + a^2)$ $\qquad 67^2 = (65 + 2)^2 = 4225 + 260 + 4) =$ **4489**

Squaring Numbers 52 to 99

$a^2 = [a - (100 - a)]100 + (100 - a)^2$ $\qquad 93^2 = (93 - 7)100 + 7^2 =$ **8649**

Squaring Numbers 101 to 148

$a^2 = [a + (a - 100)]100 + (a - 100)^2$ $\qquad 108^2 = (108 + 8)100 + 8^2 =$ **11664**

Mental Math with Tricks and Shortcuts (continued)

Squaring Numbers near 1000

	Examples
$a^2 = [a - (1000 - a)]1000 + (1000 - a)^2$	$994^2 = (994 - 6)1000 + 6^2 =$ **988036**
$a^2 = [a + (a - 1000)]1000 + (a - 1000)^2$	$1007^2 = (1007 + 7)1000 + 7^2 =$ **1014049**

Squaring Numbers that end in 1

$a^2 = (a - 1)^2 + 2a - 1$ $\quad 61^2 = 60^2 + 122 - 1 = 3600 + 121 =$ **3721**

Squaring Numbers that end in 4

$a^2 = (a + 1)^2 - (2a + 1)$ $\quad 44^2 = 45^2 - (88 + 1) = 2025 - 89 =$ **1936**

Squaring Numbers that end in 6

$a^2 = (a - 1)^2 + (2a - 1)$ $\quad 56^2 = 55^2 + 112 - 1 = 3025 + 111 =$ **3136**

Squaring Numbers that end in 9

$a^2 = (a + 1)^2 - (2a + 1)$ $\quad 79^2 = 80^2 - (158 + 1) = 6400 - 159 =$ **6341**

Using Squares to Help Multiply

Two Numbers that Differ by 1

If $a > b$ then $a \times b = a^2 - a$	$35 \times 34 = 1225 - 35 =$ **1190**
If $a < b$ then $a \times b = a^2 + a$	$35 \times 36 = 1225 + 35 =$ **1260**

Two Numbers that Differ by 2

$a \times b = [(a + b)/2]^2 - 1$ $\quad 26 \times 28 = 27^2 - 1 = 729 - 1 =$ **728**

Two Numbers that Differ by 3 ($a < b$)

$a \times b = (a + 1)^2 + (a - 1)$ $\quad 26 \times 29 = 27^2 + 25 = 729 + 25 =$ **754**

Two Numbers that Differ by 4

$a \times b = [(a + b)/2]^2 - 4$ $\quad 64 \times 68 = 66^2 - 4 = 4356 - 4 =$ **4352**

Two Numbers that Differ by 6

$a \times b = [(a + b)/2]^2 - 9$ $\quad 51 \times 57 = 54^2 - 9 = 2916 - 9 =$ **2907**

Two Numbers that Differ by an Even Number: $a < b$ and $c = (b - a)/2$

$a \times b = [(a + b)/2]^2 - c^2$ $\quad 59 \times 67 = 63^2 - 4^2 = 3969 - 16 =$ **3953**

Mental Math with Tricks and Shortcuts (continued)

Two Numbers that Differ by an Odd Number: a < b and c = [1 + (b – a)]/2

	Examples
$a \times b = (a + c)^2 - [b + (c - 1)^2]$	$79 \times 92 = 86^2 - (92 + 36) = 7396 - 128 =$ **7268**

Other Multiplying Techniques

Multiplying by 11

a x 11 = a + 10a	76 x 11 = 76 + 760 = **836**
a x 11 = If a > 9 insert a 0 between digits and add sum of digits x 10	76 x 11 = 706 + 130 = **836**

Multiplying by Other Two Digit Numbers Ending in 1 (X = 1 to 9)

a x X1 = a + X0a	63 x 41 = 63 + (40 x 63) = 63 + 2520 = **2583**

Multiplying with Numbers Ending in 5 (X = 1 to 9)

a x X5 = a/2 x 2(X5)	83 x 45 = 41.5 x 90 = 415 x 9 = **3735**

Multiplying by 15

a x 15 = (a + a/2) x 10	77 x 15 = (77 + 38.5) x 10 = **1155**

Multiplying by 45

a x 45 = 50a – 50a/10	59 x 45 = 2950 – 295 = **2655**

Multiplying by 55

a x 55 = 50a + 50a/10	67 x 55 = 3350 + 335 = **3685**

Multiplying by Two Digit Numbers that End in 9 (X = 1 to 9)

a x X9 = (X9 + 1)a – a	47 x 29 = (30 x 47) – 47 = 1410 – 47 = **1363**

Multiplying by Multiples of 9 (b = multiple of 9 up to 9 x 9)

a x b = round b up to next highest 0 multiply then subtract 1/10 of result	29 x 54 = 29 x 60 – (29 x 60)/10 = 1740 – 174 = **1566**

Multiplying by Multiples of 99 (b = multiple of 99 up to 99 x 10)

a x b = round up to next highest 0 multiply and then subtract 1/100 of result	38 x 396 = 38 x 400 – (38 x 400)/100 = 15200 – 152 = **15048**

Mental Math with Tricks and Shortcuts (continued)

SUBTRACTION

Techniques:

1) Learn to calculate from left to right: 1427 – 698 = (800 – 100) + (30 – 10) + 9 = **729**
2) Think in terms of what number added to the smaller equals the larger: 785 – 342 = **443** (left to right)
3) Add a number to the larger to round to next highest 0; then add same number to the smaller and subtract: 496 – 279 = (496 + 4) – (279 + 4) = 500 – 283 = **217** (left to right)
4) Add a number to the smaller to round to the next highest 10, 100, 1000 and etc.; then subtract and add the same number to the result to get the answer: 721 – 587 = 721 – (587 + 13) = (721 – 600) + 13 = **134**
5) Subtract a number from each number and then subtract: 829 – 534 = 795 – 500 = **295**

DIVISION

Techniques: **Examples**

Divide by parts of divisor one at a time: 1344/24 = (1344/6)/4 = 224/4 = **56**

Method of Short Division

340 ← Remainders (3, 4, and 0 during calculations)
7)1792
256 ←Answer

Divide both divisor and dividend by same number to get a short division problem

10
972/27 divide both by 9 = 3)108
36

Dividing by 5, 50, 500, and etc.: Multiply by 2 and then divide by 10, 100, 1000, and etc.

365/5 = 730/10 = **73**

Dividing by 25, 250, 2500, and etc.: Multiply by 4 and divide by 100, 1000, 10000, and etc.

Dividing by 125: Multiply by 8 and then divide by 1000

36125/125 = 289000/1000 = **289**

It can be divided evenly by:

2 if the number ends in 0, 2, 4, 6, and 8
3 if the sum of the digits in the number is divisible by 3
4 if the number ends in 00 or a 2 digit number divisible by 4
5 if the number ends in 0 or 5
6 if the number is even and the sum of the digits is divisible by 3
7 sorry, you must just try this one
8 if the last three digits are 000 or divisible by 8
9 if the sum of the digits are divisible by 9
10 if the number ends in 0

Mental Math with Tricks and Shortcuts (continued)

11 if the number has an even number of digits that are all the same: 33, 4444, 777777, and etc.
11 if, beginning from the right, subtracting the smaller of the sums of the even digits and odd digits results in a number equal to 0 or divisible by 11:

406857/11 Even = 15 Odd = 15 = 0
1049807/11 Even = 9 Odd = 20 = 11

12 if test for divisibility by 3 & 4 work
15 if test for divisibility by 3 & 5 work
Others by using tests above in different multiplication combinations

SQUARE ROOTS

Examples

Separate the number into groups of 2 digits or less beginning from the right

$(66049)^{1/2}$ 6 60 49

What number can be squared and be less than 6 = 2 with a reminder of 2
Bring down the second group of digits next to the remainder to give 260
Double the first part of the answer to get 4, divide into 26 of the 260 to get 6 as a trial number
Use 4 & 6 to get 46 and multiply by 6 to equal 276 which is larger than 260, therefore try 5
Use 4 & 5 to get 45, 45 x 5 = 225, 260 – 225 = 35, bring down the 49 to get 3549
Double the 25 to get 50, divide 50 into 354 to get 7 as a trial second part of divisor
Use 50 & 7 to get 507 and multiply 507 x 7 to get 3549 with no remainder.

See complete calculations below:

6 60 49 (**257** = Answer
4
45)260
225
507)3549
3549
0

CUBE ROOTS

Memorize the following:

Cube of 1 = **1**, 2 = **8**, 3 = **27**, 4 = **64**, 5 = **125**, 6 = **216**, 7 = **343**, 8 = **512**, 9 = **729**

Note: no result ends in the same digit

$(300763)^{1/3}$ Divide in to groups of 3 from right = 300 763

Note: the number ends in a 3. Last digit of cube will be 7 if this is a cube ***without a remainder***

Since 7 cubed = 343 and 6 cubed = 216, the left most group of 300 is between them and we must use the smaller, or 6.

The answer is **67** *This method works up to 1,000,000 for true cubes*

Mental Math with Tricks and Shortcuts (continued)

Cube Roots the Long Way

$(636056)^{1/3}$

What number cubed is less than 636 = 8. Put 8 down as first part of answer
Square 8 for 64 and multiply by 300 = 19200. Divide into 114056 = 6, add the 8 and 6 = 14
Multiply 14 x 30 = 420, add 420 to 19200 = 19620, square the 6 and add to 19620 = 19656
If 6 x 19656 is less than 124056, then it is not necessary to use lower number.

```
          636 056        = 86^3
          512
19656)124056
       117936
         7120
```

USE ESTIMATES

Use estimates to check your answers. Get in the habit of doing this for all calculations.

NOTE: Considerable care has been taken to eliminate errors in this document, but the author does not guarantee that the document is error free by implication or in fact.